



PERFUSION PUMP: MATHEMATICAL MODELING USING BOND GRAPH FOR BLOOD PRESSURE CONTROL

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ABSTRACT

Intravenous administration of medicines is usually performed using a perfusion pump. However, there are still reports of accidents associated with this technology. This article presents a Mathematical Modeling proposal based on the Bond Graph technique in order to simulate a Continuous Perfusion Pump. Presenting a mathematical model for the medicine perfusion procedure to control the mean blood pressure (MBP) is essential for the effectiveness of the treatment, which must follow a linear pattern and with a small regimen error. To this end, modern dynamic control techniques are used to know the mechanical parameters for the analysis of pressure data and to control the process through an electronically controlled pressure pump. This study has the goal of identifying the mathematical model of the pump perfusion procedure and presenting a fuzzy controller. The contribution of this study was to present the mathematical model with refinements based on prior knowledge of the phenomena involved and an analysis of dynamic control of the mathematical model with the use of fuzzy control.

INTRODUCTION

One of the biggest current challenges of science is to translate mathematical terms and relationships the operation of phenomena and systems that compose the universe. There is a wish of developing and using this knowledge of how the factors of a system relate to adapt and improve the processes that interact with such a system. This work is under the focus of Bioengineering, which presents the process of mathematical modeling of Systolic Blood Pressure (SBP), Diastolic Blood Pressure (DBP), which are inputs of the system, and the output, which is the maximum flow variable, which indicates the rate of maximum perfusion to achieve stabilization of blood pressure (BP) based on the reference value from the literature.

An example of use of perfusion therapy is for the treatment of Diabetes Mellitus, considering that Multiple Doses (MDI), as well as the Continuous Perfusion Pump for Insulin (CPP) are effective means for the treatment of Diabetes Mellitus, but the frequency and severity of hypoglycemia are higher in diabetics who use the MDI treatment when compared to those that use perfusion pump, because the pump has a smaller chance of inducing hypoglycemia, preventing future complications and improving the quality of life [1-3].

Another aspect are the risks inherent in health care; medicine perfusion is a constant concern. In [4] the types of notifications of adverse events were analyzed, with the possibility of the professionals being involved in incorrect operation of perfusion pumps (PP). The study allows us to affirm that the professionals who operate the PPs often lack knowledge and/or commitment to the activity. It can be said that an important and unquestionable aspect on the use of PP is regarding its safety [4-5].

Among the various approaches in the literature, this study adopted the Mathematical Modeling governed by the Bond Graph technique (BG), a tool for graphical representation that presents a common structure among the various physical domains, electrical quantities, mechanics, hydraulics, fluids, pneumatics, and others [15]. This tool was developed by H. M. Paytner, in 1959, who presented the revolutionary idea of portraying systems in terms of power connections, connecting the elements of the physical system to the so-called junction structures, which represented manifestations of the restrictions [9][11][15]. This description of energy exchange of a system is called Connection Graph, which can be guided by both energy and information [14].

The BG modeling was adopted because it allows for the establishment, before the finalization of the mathematical model, of the causal relations between the elements of different physical domains with the application of an algorithm (logical and consistent procedure) to obtain the equations in the state space. With the aid of the advanced simulation software 20-sim a diagram of representation of the physical phenomena was prepared in order to identify the variables of the model [10-14].



This article is intended to model and represent the electromechanical and fluid domains, as well as the skin, which compose the PP, using analogous subsystems.

MATERIALS AND METHODS

This study used Simultaneous Engineering (systematic approach for the parallel development of the design of a product and its related processes) for methodological purposes, in order to ensure that the specifications of the PP and the mathematical model do not conflict, generating a product within an incorrect specification.

Description of the perfusion pump

There are three types of systems of perfusion: (I) using a manual flow control, which is the simplest; (ii) using a perfusion controller (automatic or semi-automatic) to establish the flow determined by the operator; and (iii) the perfusion pump, which generates, monitors and controls the flow [19].

Intravenous administration of medicines is usually made using a perfusion pump. The PP is a system that offers the highest accuracy of perfusion, and allows for the use of longer sessions than gravitational systems. The perfusion pressure is independent of gravitational pressure, and it is often greater than gravitational pressure.

The PP is usually a positive displacement pump (PDP). The PDPs force the movement of fluid through variations in volume. A cavity is opened, and the fluid is admitted through an entry. The cavity then closes, and the fluid is compressed by means of an output. The PDPs have the advantage of being able to move any fluid, regardless of its viscosity [15-17].

The internal constructive form of the PP may vary, but its control system has as an input parameter engine the torque of the step motor governed by a microcontroller, and the output variables are the microdoses. The system works so that when the pump operation process starts, the step motor controls the dosage rate scheduled by the physician in accordance with the physiological needs of the patient.

Description of the catheter and cannula

The catheter consists of a tube connected with the PP and cannula. The insulin output in the Cartridge will occur with a change in the volume from the container to the connected tube (Catheter), with the purpose of allowing the flow towards the cannula, which keeps the work pressure steady [14].

The cannula is a component located in the end of the catheter that fits into the silicone connector previously attached to the skin. The entry of the flow in the body happens through capillaries located in the dermis, at the point where the cannula is connected, and the cannula causes a variation in the glucose concentration in the bloodstream.

Simplifying assumptions and parameters adopted

Some hypotheses were adopted in the modeling:

Assumptions of the Model:

Some considerations have been assumed in the modeling of the system.

- i) The volume of insulin varies according to the representation from the mechanical subsystem to the fluid subsystem.
- ii) The system parameters are concentrated.
- iii) No noise has been assumed to be in the system.
- iv) There is influence of some organs on pressure concentrations, considering that they are tissues in a state of rapid equilibrium.

Parameters:

The parameters that make up the basic elements of the system in the areas of electro mechanics, mechanics and fluids of the PP are the passive elements of a port (Resistance, Inductance and Capacitance); active elements of a port (Source of effort and flow); of two ports (Transformer and Rotor) and elements from two junctions (Junction 1 or Junction 0) [8] [10].



The theory based on Bond Graph technique consists in an approach to energy that defines two generalized power state variables (input variables effort $e(t)$ and flow $f(t)$) in order to identify the equivalent quantities in the various physical domains represented by variables of energy, and, for these variables to be indicated, the key element is a causal bar showing the necessary relationship of cause and effect between two elements [10]. This causality is indicated by a vertical bar inserted in one end of the connection pointing in the direction of the effort, and in the opposite direction the direction of the flow is defined. When establish the direction of effort and flow the connections are listed in an orderly manner, to obtain the equations through the constitutive principle of each element of the connection graph [11] [14].

The state equations are obtained by relating the power variables with the integral variables of elements I (inductance) and C (capacitance) of integral causality, associating the constitutive principle of each element of the Connection Graph, resistor elements that dissipate energy and the sources of excitation [17]. In the elements I and C the respective co-energy variables must be marked: the effort "e", identified by the derivative of the quantity of movement ($e = \dot{p}$), and the flow "f", identified by the derivative of the displacement, where $f = \dot{q}$. The number of input variables will correspond to the number of the respective sources of the system. In the elements that dissipate energy (Resistance - R, Transformers - TF and Rotors - GY) the constitutive laws must be formulated and identified; from the joints "0" and "1", taking into account the direction of power and its causalities, the state variables are listed and then replacements are made in order to obtain the state equations of the electromechanical and fluid system related to the PP [10-12].

Table 01 presents the analogues of the real passive and active elements in the BG technique. The elements are classified as dissipators, storers, transformers, rotors and energy sources [11-14].

Table 01: Analogue Of Real Variables By The Bond Graph Technique (BG).

Element	Description of actual system	Description of analogue system	Bond Graph
If	Voltage Source of stepper motor	Engine source	Effort source
B1	Natural resistance of wires from a stepper motor	Loss of voltage in the motor	Resistance
L1	Electric field generated in the coils of a stepper motor	Field generated in the coils	Inductance
A	Transfer of energy from the rotor of the stepper motor to the axis	Is the relationship of transformation	Transformer
B2	Friction of the plunger with the walls of the plunger travel	Resistance to the natural movement of the walls of the plunger travel	Resistance
M2	Total mass of the liquid inside the container	Inertance associated with the liquid	Inductance
K1	Energy losses related to plunger axis	Capacitance of the movement of the plunger axis	Capacitance
$r = 1/A2$	Translational movement of the plunger that causes pressure on the fluid	Element of reverse transformation of the translational mechanical system for fluid system	Rotor
K2	Energy loss related to the amount in the reservoir	Capacitance of the amount in the reservoir	Capacitance
B3	Resistance to the flow from the cartridge to the catheter	Fluid resistance in the passage of insulin from the cartridge to the catheter	Resistance
I3	Moving mass within the catheter	Fluid inertance within the catheter	Inductance
K3	Strength of insertion of the cannula into the individual's skin	Insertion force applied on the individual's skin	Capacitance
I4	Mass of insulin in movement from the cannula to the silicone	Inertance of the cannula-silicone assembly	Inductance
B4	Pressure loss related to variation in volume	Fluid resistance of the flexible cannula	Resistance
I5	Mass in microdoses from the silicone to the dermis	Inertance of the silicone-dermis set	Inductance
B5	Pressure loss of insulin in the passage from the silicone to the dermis	Fluid resistance of the dermis	Resistance

RESULTS

This system is obtained by the sequential application of the change from the physical model to the analogous model, in BG, from which the mathematical equations will be obtained. The BG obtained has four basic groups: passive elements of a port (Inductance or Inertance, Capacitance and Resistance); active elements of a port (Source of Effort or Flow), two ports (Transformer and Rotor) and elements of two junctions (Junction 1 or Junction 0) (Figure 1) [10]. The technique of BG modeling divides the system into subsystems, and in each subdivision, the variable of energy or power is divided into pairs: pressure-flow; strength-speed; torque-angular velocity; voltage-current. These pairs are connections between subsystems — the doors. Energy is exchanged through these ports in each element, where each port represents a single and distinct power interface [18, 19].

The formulation of the BG diagram and of the system of equations has followed the “Algorithm for Construction of Connection Graphs” supported in the use of the 20-sim software. 20-sim is an advanced simulation program that helps the graphic model, in order to design and analyze the dynamic systems; allows for modeling through block diagrams, connection graphs and fully observable equations with a structure of unlimited hierarchical model. Then, we developed a diagram representing the physical phenomena to identify the variables of the model [15].

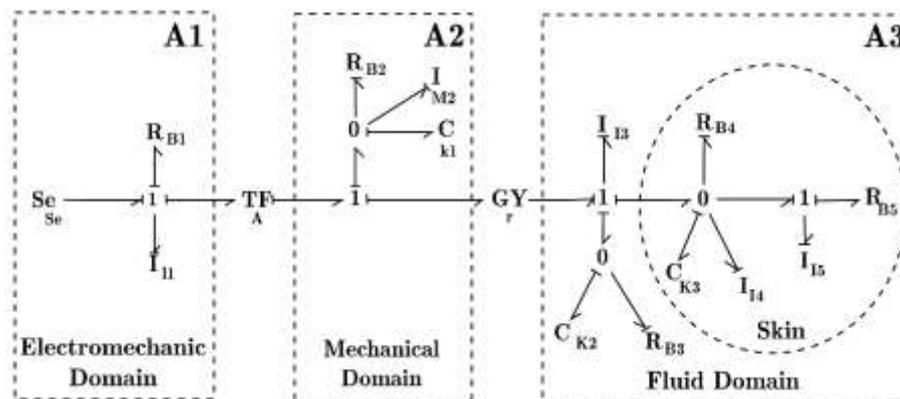


Fig. 1. Representation of the BG of the Continuous Perfusion Pump proposal

The determination of energy domains is required for the representation Continuous Perfusion Pump with the BG theory. For a better comprehension, please note that the subsystems, representing the physical domains, are identified by the regions A1, A2, A3.

Region A1: The rotor of the stepper motor is represented by the electromechanical domain. The source of effort “If” represents the voltage applied to each coil of the motor [2]. The common flow between the elements is considered as electrical current and the effort that connects to the junction 1 is considered as voltage. In the course, we have the element R, which corresponds to electrical resistance, the element “I” represents the inductance of the motor coils. The stepper motor rotor subsystem is connected to a two-port element of the “Transformer” type, which is responsible for the conversion from the electrical domain to the mechanical domain.

Region A2: The element TF has the function of converting angular velocity into linear velocity, representing in the system the change from Electrical System to Mechanical System, in order to transform the rotational movement into translational movement. The elements related to junction 1 represent energy losses related to the mechanism that drives the plunger of the insulin reservoir represented by the variable C (Capacitive element), the variable R (Resistive element that represents the resistance of the friction between the plunger and the walls of the plunger course) and the variable I (representing the total mass of the liquid in the container) [7].

Region A3: The output from the Mechanical System to the Fluid system through the element GY represents the opposite transformation, whose relationship of transformation is between the variables of input effort with exit flow and input flow with exit effort, i.e., the system shows the conversion of speed into pressure and of force into flow, represented by the connection number 11 of the fluid system, characterized by the cartridge (7). At junction 1, we can notice that there is an energy loss related to the amount present in the reservoir of the equipment and



the movement of this volume into a catheter. For the fluid system to become complete, we have considered the individual's skin, represented by the elements Capacitive (C), Inductive and Resistive, connected to junction 0. The element C represents a force of insertion of the cannula into the individual's skin; the variation of microdoses from the cannula to the silicone, element I, together with the loss insulin pressure caused by its passage through the cannula. We can conclude that there is also a resistance element, R, which suggests a variation of volume in microdoses in the skin. Finally, once again the amount in microdoses from the silicone to the dermis, element I, together with the loss of insulin pressure caused by its passage from the silicone to the body of the individual that will use it, represented by the element R, at connections of junction 1, because we can observe, in these connections, that flow is not divided.

After making the BG diagram, systematic procedures are conducted, according to the algorithm for the construction of the BG to obtain the system equations in the form of a state space, thus obtaining the mathematical modeling of the Perfusion Pump system attached to the skin. Equation 1 presents the system of equations obtained.

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ \dot{X}_5 \\ \dot{X}_6 \\ \dot{X}_7 \\ \dot{X}_8 \end{pmatrix} = \begin{pmatrix} \frac{-B_1}{L_1} & 0 & \frac{-A}{A_2 L_3} & 0 & 0 & 0 & \frac{-A}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 & 0 \\ \frac{A}{A_2 L_1} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{C_2} & \frac{-1}{C_3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_3} \\ 0 & 0 & 0 & 0 & \frac{-Rf}{L_5} & 0 & 0 & \frac{1}{C_3} \\ \frac{A}{L_1} & \frac{-1}{L_2} & 0 & 0 & 0 & \frac{-1}{C_1 B_2} & 0 & 0 \\ 0 & 0 & \frac{1}{L_3} & 0 & 0 & 0 & \frac{-1}{C_2 B_3} & 0 \\ 0 & 0 & \frac{1}{L_3} & \frac{-1}{L_4} & \frac{-1}{L_5} & 0 & 0 & \frac{-1}{C_3 B_4} \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot (V(t)) \quad (1)$$

DISCUSSION

This article describes the implementation of a commercial pump based on the construction of a mathematical model using the BG technique. To this end, the variables were evaluated in the state space in connections of integral causality, in order to obtain the equations and each diagram separately, and incorporated into each physical domain. Thus, it is necessary to reaffirm that the understanding of Modeling allows for the change and better understanding of the proposed technology, presenting in the study a simplified, but efficient pump.

With this, people with Diabetes may have a low equipment cost, giving them a better quality of life, because the innovations on the market are not within the reach of the great majority, which has to resort to traditional treatment, which, in the event of lack of metabolism adaptation, brings neurological consequences due to severe hypoglycemia, with frequent and serious future complications.

In current research, the Continuous Perfusion Pump is considered a device similar to that of the pancreas of a normal individual, because it releases ultrafast microdoses in a period of 24h, coupled to a catheter inserted in a cannula in the subcutaneous tissue, the dosages being programmed in accordance with the needs of the diabetic patient. This device prevents severe hypoglycemia, significantly improves the rates of glucose concentration, a result of the rates of glycated hemoglobin, giving the individual a life without the limitations caused by this condition. In face of this need, this study presented a new device for the treatment, more versatile and accessible. The proposal presented here is based on the presentation of a real system that allows for the development of a complex model, in order to better understand each mechanism that the electromechanical device provides when simulated in a Mathematical Model.

However, the project has presented some difficulties, several attempts being needed in order to get a more precise similarity to the device discussed in this article. Thus, in order to achieve the ideal prototype, we evaluated variables in the state space present in the connections of integral causality, and, in order to integrate each physical domain, the process took place in each diagram separately.



CONCLUSION

The model represented in this study has the objective of integrating different physical domains and correlating them in the same complex system in order to gain better understanding. For optimization of the Mathematical Modeling using the BG technique, we obtained the aid of an advanced simulation computer program, providing data on stability, transient, speed, phase space, which resulted in the improvement of the dynamic response of the device modeled. This Modeling allows for a more precise definition of the components that will be used in the assembly of the device, since each one of these components is represented in the Bond Graph Model as an element, be it passive with one port, active with one port, transducer elements with two ports or junctions 1 and 0.

With the results obtained in this study, it is possible to pursue, as a next step, the development of interfaces of the pump with other correlated devices, with biosensors or medical follow-up applications for mobile devices such as smartphones and tablets. This is feasible because both glucose meter biosensors and mobile communication devices have components that can be integrated to the mathematical modeling of the pump, using this state space approach the BG technique provides.

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